CARTOMETRIC ANALYSIS AND DIGITAL ARCHIVING OF
HISTORICAL SOLID TERRAIN MODELS USING NON-
CONTACT 3D DIGITIZING AND VISUALIZATION
TECHNIQUES.

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Abstract

This paper presents a new methodology for assessing the accuracy of historical solid terrain models using non-contact 3D digitizing techniques and demonstrates new possibilities for providing a digital archive of such precious artefacts of our cultural heritage. The famous models of the Swiss modeller Joachim Eugen Müller (1752-1833) and models created by the Allies during the Second World War have been scanned using a Minolta VI-900 laser scanner which is a high resolution, high accuracy 3D scanner that uses laser triangulation to measure distances to points. Comparison with current digital elevation data using trend surface analysis has enabled us to assess the accuracy of these models. 3D non-contact digitizing has proved itself to be highly suited to this type of research. It is fast, flexible and accurate. However, we are now in a position to suggest improvements in the way that it is operated in future. During the initial scanning it was not easy to gauge the success of each scan using the scanner’s built-in viewer, and issues with the scans only became apparent during post-processing. The texture capture proved to be especially difficult, as the necessary lighting condition for the laser scan did not lend itself for the capture of the image information. This, combined with the relatively low resolution of 640x480, meant that the images were low resolution and poorly exposed. A secondary image capture using better lighting and a better image sensor, such as a calibrated digital SLR, and subsequent image registration to the finished 3D model would be our preferred method in future. The results of the models scanned so far suggest that solid terrain models were made primarily to establish the three dimensional structure of the landscape with the emphasis on providing a depiction of the landscape as a continuously changing surface. Absolute altitudes and relative heights were perhaps of secondary importance to the more important problem of filling the gaps between known measured points and providing a
human view of the landscape. The results provide objective testimony to the skill and
devour of the model makers using primitive techniques by modern standards. The
results also demonstrate that non-contact 3D digitizing techniques not only provide a
suitable data capture method for solid terrain model analysis, but also provide a means
of preserving digital facsimiles of such precious artefacts in the future.

Historical Context

The eighteenth century witnessed significant progress in methods and techniques of
surveying and mapping in Europe. Relatively modern principles of surveying based on
triangulation had already commenced in France and Great Britain towards the end of the
century. Though admirable attempts had been made to depict the high mountains of the
Swiss Alps, most notably by Franz Ludwig Pfyffer (1716-1802) (Bürgi 2007), a
sufficiently accurate map of Switzerland, based on rigorous survey methods, did not yet
exist. Only local networks of triangulation had been established and a modern
topographical survey of the whole of the country was a distant prospect.

This deficiency was recognised by a wealthy industrialist, Johan Rudolf Meyer (1739-
1813). Meyer was an enlightened individual and, inspired by the impact of Pfyffer’s
model, invested part of his fortune in funding the first systematic survey of Switzerland.
Experienced at mountaineering, Meyer was fully aware of the challenge that lay ahead
and set about enlisting expertise to fulfil his dream. He engaged the services of the
Alsatian geometrist Johann Henry Weiss (1758-1826) and together they set about
planning their venture by ascending Titlis in the summer of 1787. A carpenter of
Engelberg, Joachim Eugen Müller, then aged 35, acted as a guide. Imhof (1981)
suggests that Meyer discovered in Müller, not only a skilful mountaineer, but also an
intelligent observer, particularly from the topographic point of view. By the following
winter, Müller had constructed a relief model of the Engelberg area and subsequently
began work for Meyer in the spring of 1788. Müller and Weiss then worked with the
mathematician and physicist, Johann George Tralles (1763-1822) of the University of
Bern during the summers of 1788 and 1789. Professor Tralles was a pioneer of modern
land surveying techniques and had begun base-line surveys in different parts of
Switzerland. No doubt Müller learned a significant amount about surveying principles
and particularly triangulation.

Survey method

The precise method of survey employed by Weiss and Müller remains unclear. The
survey however, began on 10th June 1788 whereupon Weiss, Müller and Tralles climbed
several peaks including Hohgant, Morgenberghorn, Stockhorn, and Niesen (Wolf, 1879,
cited in Klöti, 1997). This early collaboration between Meyer, Weiss, Müller and
Tralles which employed new and meticulous survey techniques did not last, Weiss and
Meyer preferring a simpler and less time-consuming triangulation technique (Klöti,
1997). Meyer funded the construction of a simple but effective surveying instrument
built by David Breitinger (1763-1834) of Zürich. Neither the instrument nor detailed descriptions of it have survived but a useful description of a contemporary instrument is provided by Imhof (1981). Imhof suggests that the instrument consisted of an alidade or diopter mounted centrally on a circular disc of wood upon which angular measurements could be plotted on a circular sheet of paper. Levelling the instrument would have been carried out using a spirit level. No angular measurements were read, a ruler attached to the alidade would be used to simply draw a line in the direction that the alidade was pointing. The vertical measurement was taken by rotating the sighting device in the vertical axis, negative or positive movement of which could be read from a scale attached to the alidade in the form of a calibrated arc. The alidade did not have a sighting telescope, simply two pins at opposite ends of the alidade. The method employed by Müller was a form of graphic triangulation, similar in some respects to plane tabling. However, the plane table is rectangular and distinctly larger than Müller’s small disc, the map’s extent being a function of the dimensions of the plane table and the scale of survey. In this way, the surveyor not only records the azimuths of the positions of points, he can also record their position through intersection in the field on the same piece of paper. Müller, on the other hand, would have used a new circular sheet of paper at each new survey station and would have determined the points of intersection either at his base camp or back at his workshop.

Imhof admits that certain things remain obscure. He suggests that it was necessary for him to have a coordinate reference system to plot his points and scale as well as the orientation. A base-line measurement was also necessary to establish the coordinate system. There was no triangulated reference system at the time and so we do not know from where Müller obtained his reference points. Imhof suggests that perhaps they were based on the preliminary work he did with Weiss and/or those of Professor Tralles. Though it is impossible to be certain of the precise technique employed by Müller, it would appear that he used a form of triangulation that required the graphical transformation of his points onto a base of a pre-defined scale. Imhof does not elaborate further on the method of plotting the intersecting points onto a base. Here, we can postulate theories based on the surviving paper discs. Perhaps plan errors could be adjusted by minor movements in the paper discs, hence the presence of slots cut along lines of measurement (Figure 1). However, this would normally require the central hole to be the same width as the slots and allow similar movement at the centre. Alternatively, the slots could have been cut to simply view the intersection of lines where two or more opaque paper discs overlap. Having established the location of the point, it would have been straightforward to mark the base underneath the discs through the coincident holes of the overlying paper discs. Indeed, this technique would have been necessary given the scale of the terrain models being made. A 15cm diameter disc would create lines 7.5cm long – or 4.5 km at a scale of 1:60,000 or 9km at a scale of 1:120,000. The slots in the disc are indeed at varying distances from the centre and not all the lines have slots within them. Note also that they are marked in ink, perhaps prior to cutting, suggesting that Müller may have been able to judge the rough distance of the target from his survey station. Given this evidence, there would appear to be little need
for a sophisticated coordinate system given the strongly graphical nature of the technique employed.

Müller would have calculated the difference in height between points by using the recorded vertical angle taken from the instrument together with the horizontal distance presumably taken from the plan plotted during the graphical triangulation process. It is unlikely that Müller could have achieved high levels of accuracy given the low precision instrument and technique he was using. We have no information that relates to the vertical datum that he used and indeed, we have no indication that the curvature of the Earth was taken into account. Furthermore, whilst we know that some 264 discs have survived (ETH Zurich, Hs 1060:469-478) we don’t know the total number of stations from which Müller surveyed or indeed the number of intersecting points that he measured.

Imhof uses various assumptions to estimate a density of one station per 200 km² which he admits is a little excessive perhaps, as this density corresponds to that of the current Swiss third order triangulation. He suggests that the number of the points obtained by intersection must be some ten times larger than that of the survey stations. Müller probably had five hundred to a thousand points for the geometrical construction of his model, assuming a surface of 20,000 km², gives a density of one point for 40 or 20 km² respectively.

Müller mapped the ground located between the points obtained on the basis of field observation and had a portable compass which enabled him to determine the orientation of valleys and mountain peaks. His panoramas demonstrate great attention to detail and would have been an essential source for modelling the terrain.
Terrain modelling method

Müller carried with him the tools required for modelling – including the plaster. At his base camp, Müller would construct small relief models of the region that he had just explored. He would then transport the models to Meyer at Aarau. While Müller conducted his own surveys and modelling, Weiss was engaged in the graphical triangulation of large expanses of Switzerland. He managed to measure altitudes of a significant amount of the country. When all this work was sufficiently advanced, Weiss and Müller (Müller practically alone, according to the opinion of Professor Rudolf Wolf [1879]) undertook the construction of a great relief during the winter months spent at Aarau. They constructed a model at a scale of 1:60,000 of the Swiss Alps and Pre-Alps regions. Weiss concentrated on the East of the country while Müller completed the rest. Towards the end of this work, Weiss developed the contents and the drawing of the map. The alpine areas were then drawn, essentially, according to this great relief. As Imhof points out, we have the very rare and interesting case of a chart established according to a terrain model, not the other way around.

Unfortunately, the relief model at a scale of 1:60,000 and a size of 1.5 by 4.5 metres does not exist anymore. Meyer had the model on display in his house in Aarau where visitors could admire its hitherto unknown representation of a major part of Switzerland. It did not take long for the French Ingénieurs Géographes in Napoleon’s service to recognise the significance of the model (Bürgi, 2007) and the Dépôt de la Guerre practically confiscated the model when it was on display in Paris. Meyer’s recompense was a fourth of the costs of the survey and construction of the model. The model was of high military importance as it showed a topographically intricate part of central Europe that had hitherto been unmapped with such accuracy. The French army therefore wanted to prevent the model falling into enemy hands. The relief model was supposedly destroyed in 1922 (Bürgi 2007).

Fortunately, Joachim Eugen Müller was a prolific builder of relief models. At least 16 different models at different scales still exist at various locations throughout Switzerland (Mair and Grieder 2006). Among them is a model of the Bernese and Wallis Alps which is not north-oriented, at a scale of approximately 1:120,000 and a size of approximately 50 by 70 centimetres. Various copies of this model have survived. It is of particular interest as it is one of the very first models made by Müller and was presented to the Bernese government in 1789 together with another model at a scale of 1:40,000 (which unfortunately does not exist anymore [Wolf, 1879]). Meyer presented these two models when applying for the permission to extend the area near lake Thun that Weiss, Tralles and Müller had previously surveyed. Meyer also applied for the permission to publish the surveyed area as part of his Atlas Suisse (Klöti 1997). Meyer was granted permission and in 1796 a first test sheet of the Atlas Suisse was published. This map named “Carte d’une partie très interessante de la Suisse” was at the scale of the latter Atlas Suisse of 1:120,000 and had the exactly same extension and orientation as the model of the Bernese and Wallis Alps described above. For further analyses, we
are therefore fortunate to have a relief model (Swiss Alpine Museum Model 420.00029) and a map that were both produced by Meyer’s team at about the same period of time and show the same geographical area.

Scanning the Models: Non-contact 3D Digitizers

Non-contact 3D digitizers are used in a wide variety of application areas including medical science for prosthesis fitting and cosmetic surgery, in manufacturing for reverse engineering and rapid prototyping and in cultural heritage for the restoration and conservation of art objects. In theory, one of the major advantages of non-contact 3D digitizers is that they operate without touching the object being digitized and therefore present minimal risk to that object. Non-contact 3D digitizers are an interesting alternative to close-range photogrammetric reconstruction (Niederöst 2003), as they are quick, easy to operate, and allow for verifying the digital model on the spot using a connected laptop computer.

The scanner used was a Minolta VI-900 laser scanner which is a high resolution, high accuracy 3D scanner that uses laser triangulation to measure distances to points. More precisely, the scanner projects a near-infrared laser stripe over objects in the scene. A camera mounted in the scanner records distortions in the shape of this stripe which is offset by a known distance from the source of the laser stripe. Minolta firmware analyses the distortions in the stripe and through triangulation converts the distortion to distance measurements (Piper et al., 2002). A digital image or texture of the scanned scene at 640x480 pixel resolution is also taken by scanning the CCD through a RGB filter while the stripe light is not emitted.

The accuracy of the scan is moderated by adjusting the focal distance. The accuracy of the scanner using the 8mm wide angle lens is given as x: +1.4mm, y: +1.04mm and z: +0.64mm.

Set up and capture

The set up of the models and the scanner is dictated by the size of the object and the physical location of and handling restrictions on the object – in our case a solid terrain model. The scanner itself is mounted on a heavy duty tripod, which allows for the scanner to be tilted sufficiently allowing scanning of a model without overhanging it, avoiding potential catastrophic damage to both the scanner and the model (Figure 2). The model is therefore scanned at an...
oblique angle and due to the nature of relief models this results in dead ground in the shadow of elevated features thus necessitating a number of scans from different angles.

A wide angle lens with a focal length of 8mm was used to allow for an object distance range of 2m. Due to the size of the models, in most cases exceeding the field of view, the models were scanned in segments. Model 420.00029 (74 x 48 cm) in the Alpine Museum, Bern was of a dimension that allowed for the model to be placed on a table. The table was then moved to allow for the scanning of multiple angles and segments of the model.

The maximum recommended ambient light for the scans is 500lx. This light level is fairly low for the subsequent capture of the colour image, resulting in images that appear dark and reproducing colour poorly. Appropriate light conditions had to be achieved by moderating blinds and lighting, thus balancing the light conditions required for the image and the laser scan.

Post-Processing

Post-processing of the scans was done using RapidForm 2002®. For each model the individual scans were imported and checked. RapidForm allows the model to be viewed with the texture or as a shaded relief (Figure 3). The first step in assembling a complete model is to register the scans or ‘shells’ to each other. The initial registration is performed by defining common points between two shells and the system then matches the overlapping areas. RapidForm takes into account the fact that pairs of corresponding points are not accurate enough while performing the command, because they are selected by the user. A secondary ‘fine’ registration automatically matches the overlapping areas and registers the shells to each other. In order to gain a measure of the success of the image registration a shell/shell deviation measurement is calculated, which provides a colour map of the deviation and a maximum error. If the error is found

![Figure 3. Post-processing of the scans using RapidForm 2002®](image-url)
to be acceptable, the two matched scans are then merged and the resulting shell used to register the next shell. The process also merges the textures of the individual shells.

The alignment of the shells in space is defined from the location of the scanner in millimetres. As the scanner is situated at an angle to the model the model space is tilted and not horizontal. A manual transformation of the finished shell was performed and the model moved into a horizontal model space by aligning the frame of the scanned model to a horizontal plane. The resulting shell was then exported as a xyz text file. Unfortunately, the merged texture, however, could not be exported satisfactorily, but can still be viewed in RapidForm.

**Resolution and Accuracy**

The VI-900 produces a point cloud with an average resolution of 0.44mm. On a flat surface the scanner produces a regular grid of points. The point density, however, is affected by the irregularity of a surface, producing increased densities on slopes that face the scanner’s laser source and having the opposite effect on slopes angled away from the scanner. Here the distances between points increase slightly to between 1mm and 1.5mm. The resulting point data set is therefore quite irregular, depending on the orientation of the terrain. The point density is still considered high enough for the purposes of this study, but will have an impact on the choice of surfacing techniques for the data.

The scanner can introduce noise at distances greater than about 1.5m. The transition is quite subtle and as the problem was not apparent whilst scanning itself a more detailed investigation into the effects of distance on the scan results is planned for the future. In the final assembly of the model, priority was therefore given to shells that displayed little noise, with additional shells only used to fill in dead ground from the high quality shells. The scans of model 420.00029 did not exhibit this problem, but other, larger, models were affected. The shell/shell deviation measurements provided by RapidForm show that the maximum error in registering the shells of model 420.00029 to each other was 1.159 mm with standard deviations between +/- 0.279mm and +/- 0.316mm for different shell combinations. These errors fall within the stated accuracy levels of the scanner hardware. The scanner provided a data set of considerable size and density. Some 797,132 data points represent the 750mm x 480mm of the model area. On average the resolution was 0.44mm which, at a scale of 1:120,000, is equivalent to about 52m ground distance.

**Georectifying the Model Data**

In order to compare the altitudes of the Müller model with modern surveyed data, the Müller data was geo-corrected in order to make the coordinate system compatible with the modern Swiss topographic survey. The supposition here is that if we eliminate plan
error as much as possible, we can then compare altitudinal differences without the extra complications of horizontal scale error, which can be looked at separately.

Georectification of the Müller model was undertaken using ERDAS Imagine®. Swisstopo Digital elevation data DHM25 re-sampled to an interval of 50m for the same geographical area as the Müller model was available as a reference for the georectification process. Control points were selected and linked for points covering the model area. Points were selected that were as unambiguous as possible, such as prominent peaks. Control point residual values were examined for error, with any point errors exceeding one pixel or greater contribution to the error being eliminated. Fifty six points remained after this process having a mean RMSE of 2.23 pixels (111.5m). An affine transformation was employed for the geo-rectification process and the final image was converted to an X, Y, and Z ASCII format file in readiness for import into Golden Software’s Surfer® software package.

Results

At this stage in our research we are mainly interested in identifying any broad trends in the differences between Müller’s model and modern surveyed points. This might throw some light on the techniques employed for the survey and indeed for the model-making process.

Much can be learnt from the analysis of the residuals. The pattern and amount of error (Figure 4) demonstrate a high degree of spatial autocorrelation throughout the model. The highest errors appear to be located on the central portion of the model, the Bernese Oberland. When we consider both positive and negative residuals we can see that the distribution of error is not random, with overestimations of height being evident in the north-west and south of the model. The central Bernese Oberland appears to be significantly underestimated. In order to examine the general trends in the error, trend surface analysis was applied to the residual values. A first order polynomial interpolation of

![Figure 4 Positive and negative residuals](image-url)
the residuals suggests a systematic error that follows the broad trend described above. Apart from isolated peaks in the south, the broad trend is overestimation towards the north of the model and a dip towards the south. Using the polynomial surface, we can adjust the Müller data accordingly. The overall impression from these adjusted errors is that the central Bernese Oberland at the centre of the model has been underestimated. A quadratic polynomial trend surface was then applied to the adjusted surface’s residuals and an adjustment made once again. The pattern of the residuals demonstrates a much lower level of spatial autocorrelation with high residual values concentrated on valley slopes rather than mountain peaks and valleys.

Another interesting angle to take is to compare the relative altitudes of the principal mountains with the model area (Figure 5). This is very revealing. Even over very short distances, Müller’s estimation of altitude appears to be at odds with reality. This is also not limited to the Bernese Oberland. Interestingly, when we compare the rank order of mountain altitudes as depicted on the Meyer-Weiss Atlas de Suisse map of 1797 we find that the rank order here is in harmony with today’s data.

As with all analysis of this nature, any explanation for the distribution of error between Müller’s model and the modern surveyed data supplied by Swisstopo will be largely educated guesswork. There are many factors that could contribute to the results as outlined so far; some of them not associated with Müller’s survey and model construction techniques. These factors include the errors introduced by the 3D digitizer and the deformation of the terrain model over time. We must also be mindful that the model may not have been mounted in its frame in its original horizontal position. Furthermore, as there is no datum identifiable on the model, an arbitrary datum had to be applied to the scanned data. When we consider that the maximum range of height within the model is a mere 4cm or so, these factors may well have had a significant impact on the results.

Nevertheless, even at this exploratory stage, we can establish some working hypotheses with which we can move forward in our research. Firstly, the models appear to have been constructed by establishing principle peaks in positions that compare closely to today’s data. Intervening surfaces were perhaps modelled by ‘interpolation’ as one would expect, to a lower level of accuracy in both height and plan. These principle peaks do not appear to have been modelled to the same level of accuracy in Z as they...
have in X and Y, the emphasis being focussed towards plan accuracy rather than height. The lower accuracy in the vertical axis of the model is not likely to have been due to inaccuracies in surveyed height data as contemporary values, available to Müller (as seen on the Atlas Suisse), were closer to reality.

**Conclusions**

This paper has attempted to apply scientific and objective measures in assessing the models of Joachim Eugen Müller and of necessity has had to forgo any appreciation of the exceptional levels of landscape modelling that were achieved. Throughout the paper, the term ‘error’ has been used frequently in its statistical sense but still might give the impression that the analysis has been focussed on weaknesses rather than strengths. However, it is abundantly clear that the models are a remarkable testimony to the dedication, skill and artistry of Joachim Eugen Müller. Given the lack of sophisticated surveying equipment and established triangulated survey network, his achievements are all the more remarkable. Indeed, looking at the models, it is difficult to believe that they are some 200 years old. The high standard of his work helped to establish a new benchmark in Swiss cartography and indeed was the progenitor of a Swiss ‘school’ of modelling (see the excellent book by Mair and Grieder, 2006).

3D non-contact digitizing has proved itself to be highly suited to this type of research. It is fast, flexible and accurate. However, we are now in a position to suggest improvements in the way that it is operated in future. During the initial scanning it was not easy to gauge the success of each scan using the scanner’s built-in viewer, and issues with the scans only became apparent during post-processing. The texture capture proved to be especially difficult, as the necessary lighting condition for the laser scan did not lend itself for the capture of the image information. This, combined with the relatively low resolution of 640x480, meant that the images were low resolution and poorly exposed. A secondary image capture using better lighting and a better image sensor, such as a calibrated digital SLR, and subsequent image registration to the finished 3D model would be our preferred method in future.

The results of our analysis suggest that the terrain models were made to establish the three dimensional structure of the landscape with the emphasis on providing a depiction of the landscape as a continuously changing surface. Absolute altitudes and relative heights were perhaps of secondary importance to the more important problem of filling the gaps between known measured points. With Müller’s talent for landscape recording, he ensured that the first ‘modern’ maps of Switzerland depicted its landscape as closely as possible and ahead of the systematic triangulated surveys that commenced later in the nineteenth century.
References


