

A compromise aspect-adaptive cylindrical projection for world maps

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There are two problems with current cylindrical projections for world maps. First, existing cylindrical map projections have a static height-to-width aspect ratio and do not automatically adjust their aspect ratio in order to optimally use available canvas space. Second, many of the commonly used cylindrical compromise projections show areas and shapes at higher latitudes with considerable distortion. This article introduces a new compromise cylindrical map projection that adjusts the distribution of parallels to the aspect ratio of a canvas. The goal of designing this projection was to show land masses at central latitudes with a visually balanced appearance similar to how they appear on a globe. The projection was constructed using a visual design procedure where a series of graphically optimized projections was defined for a select number of aspect ratios. The visually designed projections were approximated by polynomial expressions that define a cylindrical projection for any height-to-width ratio between 0.3:1 and 1:1. The resulting equations for converting spherical to Cartesian coordinates require a small number of coefficients and are fast to execute. The presented aspect-adaptive cylindrical projection is well suited for digital maps embedded in web pages with responsive web design, as well as GIS applications where the size of the map canvas is unknown a priori. We highlight the projection with a height-to-width ratio of 0.6:1, which we call the Compact Miller projection because it is inspired by the Miller Cylindrical projection. Unlike the Miller Cylindrical projection, the Compact Miller projection has a smaller height-to-width ratio and shows the world with less areal distortion at higher latitudes. A user study with 448 participants verified that the Compact Miller – together with the Plate Carrée projection – is the most preferred cylindrical compromise projection.

Keywords: aspect-adaptive cylindrical projection; Compact Miller projection; Miller projection; adaptive composite map projections; Mercator; Flex Projector

1. Introduction

Cylindrical projections with equatorial aspect show meridians and parallels as parallel straight lines. The projected parallels and meridians intersect at right angles, and the world is mapped to a rectangle. Many cartographers do not recommend using cylindrical projections for mapping the world because of the notion that rectangular world maps mislead the map user's interpretation of the world's shape. Despite resounding opposition to the use of cylindrical projections for world maps, the Mercator projection has become the most frequently used projection for web maps in recent years, regardless of a 'long history of discussion about its inappropriateness for general-purpose mapping, particularly at the global scale' (Battersby *et al.* 2014, p. 85).

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We identify three arguments for the use of cylindrical projections: (1) certain phenomena that change with longitude are presumably easiest to read on a map with straight meridians, such as a map showing world time zones, (2) the appearance of land masses on cylindrical map projections is familiar to all map users due to the widespread use of the Web Mercator projection, and (3) the rectangular shape of cylindrical projections considerably simplifies page layouts. Additionally, rectangular projections seem to be preferred by some map-makers; however, it is unclear whether this is due to indifference, ignorance or simply aesthetic preference. In 1993, before the widespread use of the cylindrical Mercator projection for web maps, Werner studied projection preferences of map users and found that projections with round shapes were preferred to maps with rectangular outlines (Werner 1993). However, Šavrič *et al.* (in press) found in a recent user study that the cylindrical Plate Carrée is one of the most preferred projections among nonexpert map users when comparing nine commonly used map projections for world maps.

This article introduces a new compromise cylindrical map projection that adjusts the distribution of parallels to the aspect ratio of the canvas. The following introductory sections identify the rationale for developing another map projection, discuss existing transformations that result in cylindrical projections with varying aspect ratios and identify candidate projections for constructing an aspect-adaptive cylindrical projection. The ‘Methods’ section discusses the process for visually designing the members of the aspect-adaptive cylindrical projection family and describes the method used for converting the visually designed projections to a polynomial expression. The first ‘Result’ section describes the aspect-adaptive map projection for aspect ratios between 0.3 and 1. The second ‘Result’ section describes the Compact Miller projection, a member of the aspect-adaptive projection family with an aspect ratio of 0.6. The evaluation is split into two sections. The first ‘Evaluation’ section analyses distortion properties of the proposed aspect-adaptive projections. The second ‘Evaluation’ section discusses the Compact Miller projection in further detail.

1.1. Rationale for the aspect-adaptive cylindrical map projection

We developed a new family of cylindrical map projections that adjust the distribution of parallels to the aspect ratio of the map for two reasons. First, modern web pages use responsive web design that adapts the layout to the viewing environment by using flexible-sized text, vector graphics and raster images. Adapting a map to available canvas space in such responsive layout systems is desirable in order to use available space efficiently, particularly on mobile devices with small displays. Projections adjusting their aspect ratios to available screen space could also be beneficial when integrated with a desktop or Web-based GIS where screen size is unknown a priori or where the size of windows changes frequently. We also expect aspect-adaptive cylindrical projections to simplify the workflow for print cartography, where a map’s dimensions often need to be adjusted to the available size determined by the page layout.

The second reason for developing a new projection stems from our dissatisfaction with available cylindrical compromise projections. This may be surprising considering the hundreds of map projections invented by cartographers in the past. For example, John P. Snyder’s (1993) seminal inventory of the history map projections lists 265 major projections, but it could be extended further with less commonly used projections. However, the number of cylindrical projections in Snyder’s inventory is relatively

small. Only 39 of the 265 projections are cylindrical projections. Of those 39 projections, 12 are for large-scale maps based on ellipsoids and are not useful for world maps, 4 projections are specialized projections for mapping satellite tracks and similar applications, 5 projections are transverse or oblique variants and therefore not useful for world maps with equatorial aspect, and the central cylindrical projection is only useful for didactical purposes due to its gross area distortion. Of the remaining 17 cylindrical projections, one is the conformal Mercator, one is the equal-area Lambert cylindrical with three variations (Gall-Peters, Behrmann, and Trystan Edwards), one is the equirectangular with a variation by Gall, and five are variations of perspective cylindrical projections (Gall, Braun, Braun's second, Kamenetskiy and BSAM [after the Bolshoi Sovetskii Atlas Mira or the Great Soviet World Atlas]). Only six cylindrical projections designed for world maps using other approaches remain in Snyder's list. The six projections are the Pavlov, Miller Cylindrical, Arden-Close, Kharchenko-Shabanova, Kavrayskiy I and Urmayev Cylindrical III projections.

Five additional compromise cylindrical projections for world maps that are not listed by Snyder (1993) and not members of the equal-area, equirectangular or perspective projection families can be found in other cartographic literature. These include the Miller Perspective Compromise, Miller II, Urmayev Cylindrical II, Tobler Cylindrical I and Tobler Cylindrical II projections. Table 1 orders the 11 cylindrical compromise projections by increasing aspect ratio; 5 of the 11 projections in Table 1 have aspect ratios greater than 0.8 and were designed to compensate for the huge polar distortion of the Mercator projection. However, projections with such high aspect ratios should be avoided if possible because of their tendency to grossly distort areas and shapes. Figure 1 shows the projections listed in Table 1 with aspect ratios between 0.5 and 0.8, as well as the Plate Carrée and Braun Stereographic projections.

We could not identify additional compromise cylindrical projections with aspect ratios between 0.5 and 0.8 (excluding variations of equirectangular and perspective projections). The reason for this shortage may be that 'because of the very simplicity of cylindrical projections in the normal aspect, they were generally ignored by mathematicians and the more scientific map-makers especially attracted to the development of new projections' (Snyder 1993, p. 104). With the discovery of the only

Table 1. Cylindrical compromise projections ordered by aspect ratio. Equirectangular, stereographic and equal-area cylindrical projections are not included.

Projection	Aspect ratio	Reference
Pavlov	0.421	Graur (1956), Snyder (1993)
Miller Perspective	0.543	Miller (1942)
Miller II	0.629	Miller (1942)
Urmayev Cylindrical II	0.698	Bugayevskiy and Snyder (1995)
Tobler Cylindrical I	0.706	Tobler (1997)
Miller Cylindrical	0.733	Miller (1942)
Arden-Close	0.803	Arden-Close (1947), Snyder (1993)
Tobler Cylindrical II	0.832	Tobler (1997)
Kharchenko-Shabanova	0.838	Maling (1960)
Kavrayskiy I	0.877	Graur (1956), Snyder (1993)
Urmayev Cylindrical III	0.922	Maling (1960)

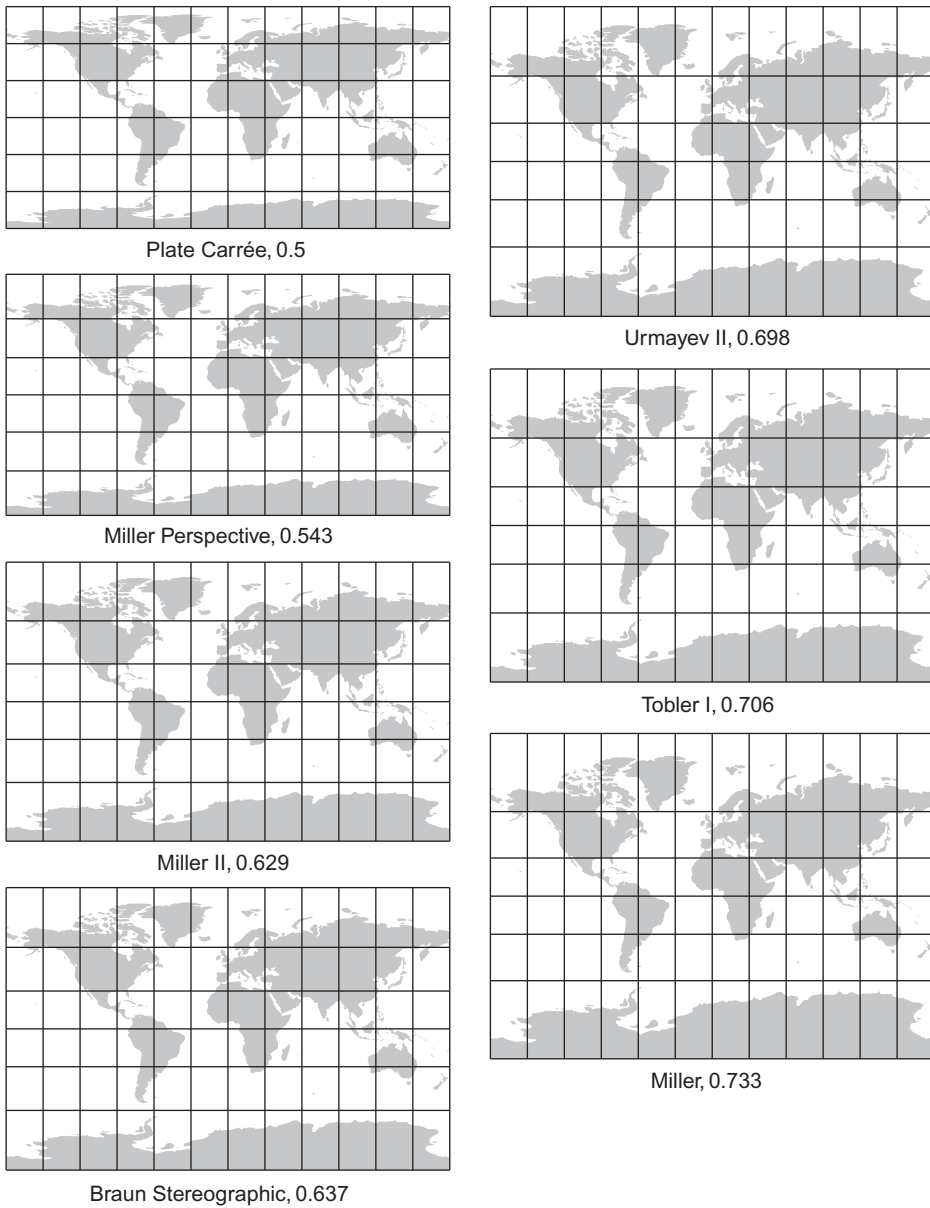


Figure 1. Compromise cylindrical projections with aspect ratios between 0.5 and 0.8.

conformal cylindrical projection (Mercator) and the two families of equal-area and equidistant cylindrical projections, only compromise cylindrical projections can be devised. To customize these compromise cylindrical projections, parallels are distributed in different ways. Among the few who customized cylindrical projections are various Soviet cartographers (see Maling 1960 for an overview), Osborn Maitland Miller (1942; see Monmonier 2002 for the development of Miller's projection) and Waldo R. Tobler (1997).

1.2. Transformations for compromise cylindrical projections

The main objective when developing the aspect-adaptive cylindrical projection was to adjust the distribution of parallels to the aspect ratio of the map in a visually balanced way. Other transformable compromise cylindrical projections that change their aspect ratio and adjust the distribution of parallels are reviewed below. These transformable projections served as inspirations for the aspect-adaptive cylindrical projection.

The equirectangular projection is an example of a transformable projection. The aspect ratio can be adjusted by changing the standard parallel. However, at larger aspect ratios – created with a standard parallel of 35° or higher – the equirectangular projection stretches land masses in a visually disturbing way. The cylindrical stereographic projection, an alternative projection where the aspect ratio can also be adjusted by varying the standard parallel, has less drastic vertical stretching at large aspect ratios.

The Miller Cylindrical and the Miller II projections, developed by Miller (1942), are two configurations of a continuous series of compromise cylindrical projections constructed by adding two terms (m and n) to the Mercator equation. By varying the values of the two terms, a variety of cylindrical compromise projections can be constructed. The limiting cases are the equirectangular projection on the one end and the (optionally stretched or compressed) Mercator projection on the other. Miller's projection series shows areas close to the equator similar to the Mercator projection, which is a pleasing characteristic to many cartographers and has made the Miller Cylindrical a popular choice among map-makers.

Like Miller's transformed Mercator, Canters (2002, p. 60) suggested modifying the cylindrical equal-area projection. The equirectangular projection is also the limiting case for this transformation. Miller's and Canters' transformations can be combined by, for example, choosing the Plate Carrée projection as the pivotal projection. Miller's transformation is applied to maps with aspect ratios greater than 0.5, and Canters' transformation is applied to maps with aspect ratios less than 0.5. The result is a smooth transition between the Mercator projection for maps with an aspect ratio 1:1, and the Lambert equal-area cylindrical projection for maps with an aspect ratio of 0.318. For both projections, the m and n parameters can be computed for a given aspect ratio using the Newton–Raphson method.

The drawback of the equirectangular, the stereographic, Miller's transformed Mercator and Canters' transformed Lambert projection families is that the polar areas are excessively stretched or enlarged at larger aspect ratios. These projections are therefore not viable options for an aspect-adaptive cylindrical projection.

1.3. Candidate projections for constructing the aspect-adaptive cylindrical projection

The cylindrical projections with variable aspect ratios discussed in the previous section all have shortcomings. In order to identify candidate projections for inclusion in the new aspect-adaptive cylindrical projection, this section examines existing projections and evaluates their potential for inclusion.

The described combination of Miller's transformed Mercator projection and Canters' transformed Lambert cylindrical projection uses the Plate Carrée as a pivotal projection (Figure 1, top left). The Plate Carrée is a reasonable choice because of its simplicity, equidistance property along meridians and low linear scale distortion, and many map users are likely familiar with this projection. The Plate Carrée is the standard projection for disseminating raster data-sets of the Earth, and many maps employ the Plate Carrée 'as

is' with no additional projection transformations. Šavrič *et al.* (in press) found that map users prefer the Plate Carrée projection over eight other commonly used projections for world maps. Prevalence, convenience and user preference aside, the Plate Carrée is not ideal for general mapping because of the horizontal stretching it applies to higher latitudes.

The Miller Perspective projection (which is not constructed with Miller's Mercator transformation described in Section 1.2) shows the world with an aspect ratio of 0.543 (Figure 1). This relatively compact aspect ratio compresses both middle and high latitudes, giving land masses a shortened appearance. Similar alternative projections include Braun Stereographic, a cylindrical stereographic projection that uses the equator as the standard parallel (Figure 1, bottom left), and Miller II. Although both of these projections improve upon the original Miller projection, polar areas are still too large compared to the mid-latitudes and tropics. The Urmayev II and Tobler I projections – both with aspect ratios close to 0.7 – are similar in appearance to the Miller Cylindrical. The Tobler I projection was developed as a computationally more efficient alternative to the Miller projection. Although both Urmayev II and Tobler I projections are more compact and devote slightly less space to polar areas, they are fairly unknown and unavailable in most mapping software (Figure 1, top right).

In our opinion, the projections in Figure 1 are the best available compromise cylindrical projections for a variety of aspect ratios up to now. However, these projections generally dedicate too much canvas space to polar areas, considerably inflating the area of higher latitudes, or apply too much compression to the mid-latitudes. For example, southern South America looks unusually short in the Miller Perspective and Plate Carrée projections.

2. Methods

To visually design the aspect-adaptive cylindrical map projection, eight different cylindrical projections with aspect ratios between 0.3:1 and 1:1 were produced using a customized version of Flex Projector (Jenny *et al.* 2010, 2013). Flex Projector is a free, cross-platform application for creating custom world map projections. The customized version of Flex Projector differed from the standard version in that the user can lock the aspect ratio for the projection that is designed. We adjusted, visually assessed and corrected the vertical distances of parallels for the eight different projections with aspect ratios between 0.3:1 and 1:1 until no further improvement seemed possible. Each of the eight visually designed cylindrical projections is defined by 18 vertical distances, one distance for every 5° of latitude between the equator and one pole. With eight projections and 18 values for every projection, there are a total of 144 values. Coding equations with these many values is impractical because of the increased likelihood of typographical errors. Additionally, programmers would need to develop code to interpolate between vertical distances, which would result in incompatible implementations if different interpolation methods were applied. This is an issue for the Robinson projection, for example, which is defined by two sets of numbers specifying the length and vertical distribution of parallels. As Šavrič *et al.* (2011) point out, there are various incompatible interpolating and approximating methods. Therefore, we used the least squares method to develop a polynomial expression for the aspect-adaptive cylindrical projection (discussed in Section 3.1). The numerical values defining the vertical distances of parallels for the eight projections served as the basis for developing this polynomial expression. The polynomial expression provides the

vertical distance of parallels from the equator for any given latitude and an aspect ratio between 0.3:1 and 1:1.

Cylindrical projections, including projections by Pavlov, Urmayev, Kharchenko-Shabanova and Tobler (see Table 1 for references), have also been defined using polynomials. Šavrič *et al.* (2011) provide details about creating polynomial expressions for projections designed with Flex Projector using the Natural Earth projection as an example. Similar to the aspect-adaptive cylindrical projection, the Natural Earth projection was first visually designed with Flex Projector, and then, polynomial equations were developed using the method of least squares.

The goal when developing polynomial equations for the aspect-adaptive cylindrical projection was to simplify the mathematical model to reduce the number of required parameters and to simplify the programming of projection equations. Various polynomial forms with different degrees were compared. The weighted least square adjustment method was used, and weights were adjusted in a trial-and-error procedure until the resulting expression closely approximated the eight visually designed projections.

To evaluate the distortion properties of the aspect-adaptive cylindrical projection, the weighted mean error in areal distortion, the weighted mean error in the overall scale distortion and the mean angular deformation were computed for a number of aspect ratios using Equations (1), (2) and (3) (Canters and Declair 1989, Canters 2002).

$$D_{ab} = \frac{1}{S} \sum_{i=1}^k \left(\frac{a_i^q + b_i^r}{2} - 1 \right) \cos \phi_i \Delta \phi \Delta \lambda \quad (1)$$

$$D_{an} = \frac{1}{S} \sum_{i=1}^k 2 \arcsin \left(\frac{a_i - b_i}{a_i + b_i} \right) \cos \phi_i \Delta \phi \Delta \lambda \quad (2)$$

$$D_{ar} = \frac{1}{S} \sum_{i=1}^k ((a_i b_i)^p - 1) \cos \phi_i \Delta \phi \Delta \lambda \quad (3)$$

where D_{ab} is the weighted mean error in the overall scale distortion, D_{an} is the mean angular deformation, D_{ar} is the weighted mean error in areal distortion, a_i and b_i are the maximum and minimum scale distortions at the sample point, $S = \sum_{i=1}^k \cos \phi_i \Delta \phi \Delta \lambda$ is the sum of the area weight factors, ϕ_i is the sample point latitude, $\Delta \phi$ and $\Delta \lambda$ are intervals in the latitude and longitude (2.5° for all computations in this article), k is the number of sample points, and p , q and r coefficients are defined as

$$p = \begin{cases} 1 & a_i b_i \geq 1 \\ -1 & a_i b_i < 1 \end{cases}, \quad q = \begin{cases} 1 & a_i \geq 1 \\ -1 & a_i < 1 \end{cases}, \quad r = \begin{cases} 1 & b_i \geq 1 \\ -1 & b_i < 1 \end{cases}$$

The three indices described above are the most commonly used measures in map projection literature and are applied, for example, by Snyder (1987, 1993), Canters and Declair (1989), Canters (2002) and Šavrič and Jenny (2014).

3. Results

3.1. Result 1: the aspect-adaptive cylindrical map projection

Figure 2 shows the aspect-adaptive projection family for height-to-width aspect ratios between 0.3:1 and 1:1. The highest aspect ratios (close to 1) resemble the Mercator projection, but slightly reduce the extreme area distortion close to the poles. The projection at the 0.6 aspect ratio is the Compact Miller, which is discussed in the following section. The Plate Carrée projection is used for the aspect ratio of 0.5 for reasons outlined in Section 1.3. The lowest aspect ratio of 0.3 produces a map that is nearly equal-area with highly flattened polar land masses.

Equation (4) defines the aspect-adaptive cylindrical map projection for aspect ratios between 0.3:1 and 0.7:1. This is a polynomial surface with 12 polynomial terms determined with the method described in the previous section.

$$x = \lambda \text{ and } y_1(\varphi, \alpha) = \frac{\varphi \cdot k_1 + \varphi^3 \cdot k_2 + \varphi^5 \cdot k_3}{n} \cdot \alpha \cdot \pi \quad (4)$$

where x and y_1 are the projected coordinates, φ and λ are the latitude and longitude, α is the height-to-width aspect ratio of the map canvas, $k_1 = A_1 + A_2 \cdot \alpha + A_3 \cdot \alpha^2 + A_4 \cdot \alpha^3$, $k_2 = A_5 + A_6 \cdot \alpha + A_7 \cdot \alpha^2 + A_8 \cdot \alpha^3$ and $k_3 = A_9 + A_{10} \cdot \alpha + A_{11} \cdot \alpha^2 + A_{12} \cdot \alpha^3$, and n is a normalization factor. The values of the 12 polynomial coefficients A_i are given in Table 2. The coefficients k_1 , k_2 and k_3 are independent of φ and λ and they can be precomputed during the initialization of the projection when the aspect ratio α is known. After initialization, they can be treated as constants. The same is the case for the normalization factor n , which is equal to $\varphi_p \cdot k_1 + \varphi_p^3 \cdot k_2 + \varphi_p^5 \cdot k_3$ with $\varphi_p = \pi/2$. The computational cost per point consists of four multiplications and two additions when the polynomial for y in Equation (4) is factorized as in Equation (5):

$$x = \lambda \text{ and } y_1(\varphi, \alpha) = \varphi \cdot (\overline{k_1} + \varphi^2 \cdot (\overline{k_2} + \varphi^2 \cdot \overline{k_3})) \quad (5)$$

with $\overline{k_i} = k_i \frac{\alpha\pi}{n}$.

For the aspect ratio between 0.7:1 and 1:1, the vertical y coordinate of the aspect-adaptive cylindrical map projection is computed in two steps: (1) the y_1 coordinate is computed with Equation (5) using the aspect ratio 0.7:1, and (2) y_1 for areas above 45° N and below 45° S is modified by adding a four-term polynomial (Equation (6)).

$$y_2(\varphi, \alpha) = y_1(\varphi, 0.7) + \tilde{\varphi} \cdot (\overline{k_{2,1}} + \tilde{\varphi}^2 \cdot \overline{k_{2,2}}) \quad (6)$$

$$\tilde{\varphi} = \begin{cases} \varphi - \frac{\pi}{4}, & \varphi > \frac{\pi}{4} \\ 0, & |\varphi| \leq \frac{\pi}{4} \\ \varphi + \frac{\pi}{4}, & \varphi < -\frac{\pi}{4} \end{cases}$$

where $y_2(\varphi, \alpha)$ is the y coordinate for the aspect ratio greater than 0.7, φ is the latitude, α is the aspect ratio of the map canvas, $y_1(\varphi, 0.7)$ is the y coordinate computed with Equation (5) and an aspect ratio of 0.7, $\overline{k_{2,i}} = k_{2,i} \frac{(\alpha-0.7)\pi}{n_2}$, where $k_{2,1} = B_1 + B_2 \cdot \alpha$, $k_{2,2} = B_3 + B_4 \cdot \alpha$, and n_2 is a normalization factor equal to $\pi/4 \cdot k_{2,1} + (\pi/4)^3 \cdot k_{2,2}$. The values of the four polynomial coefficients B_i are given in Table 3. The additional

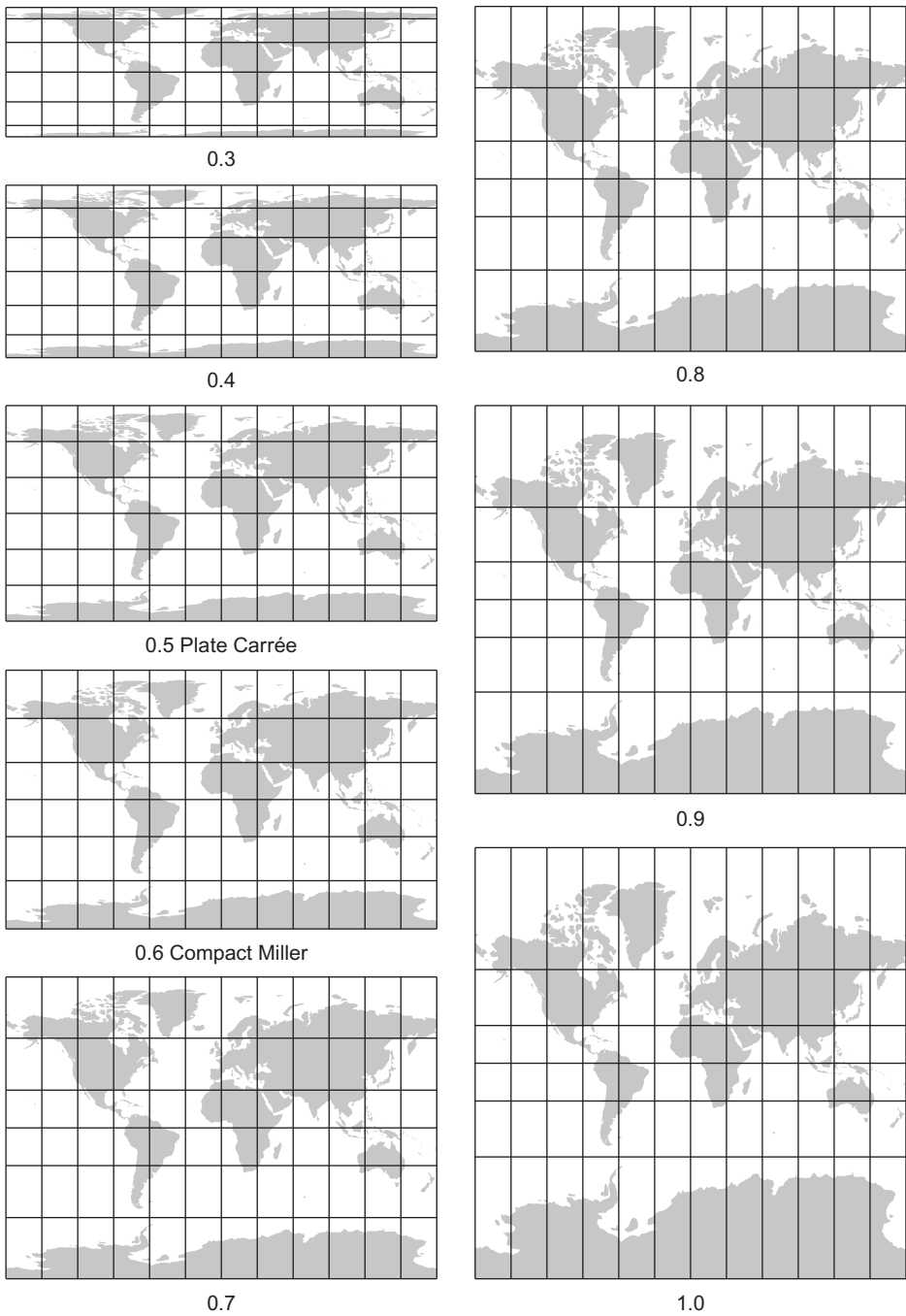


Figure 2. The aspect-adaptive cylindrical projection for aspect ratios between 0.3:1 and 1:1.

Table 2. Polynomial coefficients A_i for Equations (4) and (5).

Coefficient k_1		Coefficient k_2		Coefficient k_3	
A_1	9.684	A_5	-0.569	A_9	-0.509
A_2	-33.44	A_6	-0.875	A_{10}	3.333
A_3	43.13	A_7	7.002	A_{11}	-6.705
A_4	-19.77	A_8	-5.948	A_{12}	4.148

Table 3. Polynomial coefficients B_i for Equation (6).

Coefficient $k_{2,1}$		Coefficient $k_{2,2}$	
B_1	0.0186	B_3	-1.179
B_2	-0.0215	B_4	1.837

computational cost per point above or below the $\pm 45^\circ$ parallel consists of three multiplications and two additions when the $\overline{k_{2,i}}$ parameters are precomputed.

Figure 3 shows the polynomial surface defined by Equations (5) and (6). The vertical axis shows the vertical distance of parallels to the equator. For a more accurate comparison, the distances defined by Equations (5) and (6) have been divided by $\alpha \cdot \pi$, which reduces distances to the range $[0 \dots 1]$. Figure 4 shows the distance of parallels to the

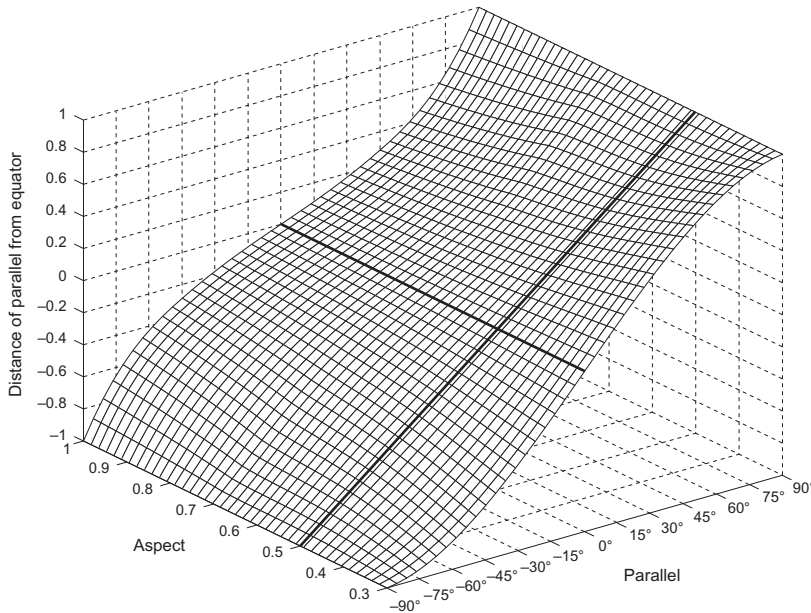


Figure 3. Polynomial surface defined by Equations (5) and (6) with distances normalized to $[0 \dots 1]$. The equator and aspect ratio of 0.5 (Plate Carrée) are highlighted.

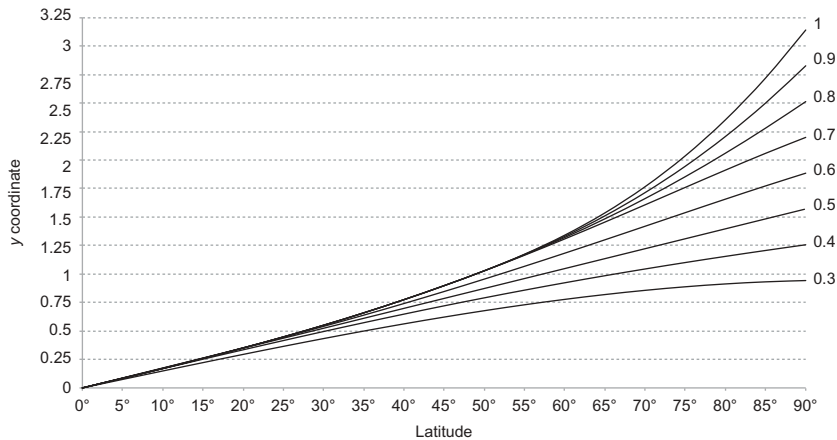


Figure 4. Distance of parallels to the equator for aspect ratios between 0.3:1 and 1:1.

equator for selected aspect ratios between 0.3:1 and 1:1. For the aspect ratio of 0.5:1, the distribution is linear and the resulting projection is the Plate Carrée. For aspect ratios less than 0.5:1, the slopes of the curves indicate a gradual decrease of spacing between parallels towards the poles. For aspect ratios greater than 0.5:1, the curves bend in upward direction, indicating an increasing spacing between parallels. When designing the polynomial expression, care was taken to obtain curves that gradually diverge towards higher aspect ratios to avoid sudden discontinuities in the graticule.

A single polynomial equation cannot sufficiently approximate all reference projections because the distributions of parallels between 55° N and S are almost identical for aspect ratios between approximately 0.7 and 1. This phenomenon is visualized in Figure 4 by the curves for aspect ratios 0.7, 0.8, 0.9 and 1. At lower latitudes, the four curves overlap; the curves begin diverging at latitude 45°. When developing equations for the aspect-adaptive cylindrical projection, it was therefore necessary to extend the polynomial in Equation (5) with Equation (6) for higher aspect ratios despite extensive trials with different polynomial degrees for Equation (5) and adjusting weights for the least square adjustment for selected aspect ratios and latitude ranges.

3.2. Result 2: the Compact Miller projection

The Compact Miller projection is a particular case of the aspect-adaptive cylindrical projection family with a 0.6 aspect ratio (Figure 5). The Compact Miller has been carefully designed because it is likely to be used frequently for mapping due to its favourable aspect ratio. We presume that many professional cartographers would select this aspect ratio when asked to choose a cylindrical projection because it is relatively close to the natural 1:2 ratio between the length of meridians and the equator, and it shows land masses in a balanced manner.

The Compact Miller preserves the shape of equatorial and mid-latitude land masses found on the Miller Cylindrical, which is a familiar projection to many users. Higher latitudes are shown with a compromise between minimizing areal exaggeration and retaining the characteristic shapes of land masses, such as Greenland. When designing the Compact Miller projection, it was important that the distance between lines of latitude

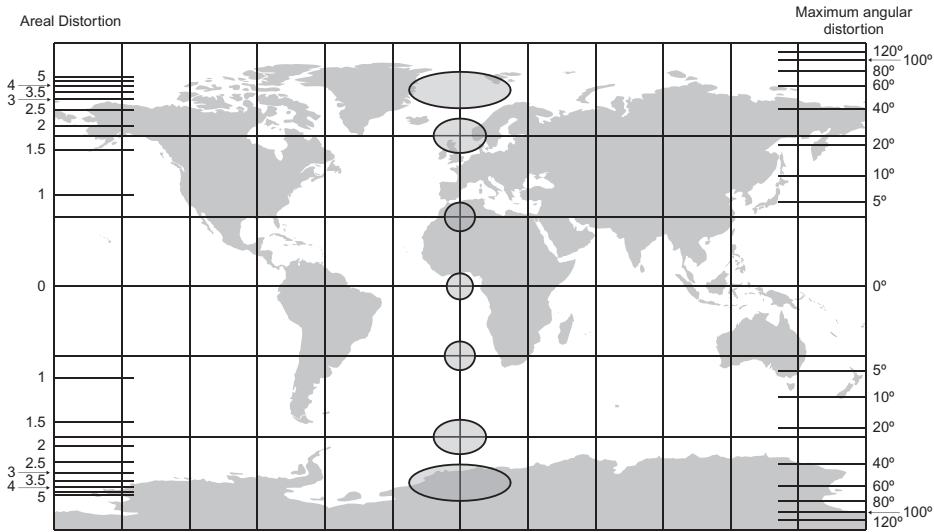


Figure 5. Compact Miller projection with areal (left) and maximum angular distortion isolines (right) and Tissot indicatrices (centre).

did not decrease towards the poles. The Compact Miller projection duplicates the spacing of parallels from the Miller Cylindrical projection between latitude 55° N and S. From latitude 55° N and S to the poles, the Compact Miller projection maintains approximately constant spacing. This construction principle is similar to the composite projection proposed by Kavrayskiy for his first projection (Snyder 1993). Kavrayskiy used the Mercator projection between latitudes 70° N and S and the equirectangular projection beyond these latitudes (for equations, see Jenny and Šavrič *in press*).

Equation (5) defines the Compact Miller projection with $\bar{k}_1 = 0.9902$, $\bar{k}_2 = 0.1604$ and $\bar{k}_3 = -0.03054$.

4. Evaluation

4.1. Evaluation of the aspect-adaptive cylindrical projection

The aspect-adaptive cylindrical projection smoothly adjusts the distribution of parallels when the aspect ratio changes. The aspect-adaptive cylindrical projection shows the central equatorial part with relatively small distortion, but accepts a compromise in higher latitudes where shape and area distortions are larger. We consider aspect ratios between 0.55 and 0.70, a good compromise between acceptable areal distortion at high latitudes and an overall pleasing appearance. Cylindrical maps with aspect ratios beyond this range either introduce disproportionate areal distortion or distort the shape of map features considerably. Deciding on a specific aspect ratio within the 0.55–0.70 range depends on the map's purpose, available space for the map in a graphical layout and personal taste. As the aspect ratio increases from 0.70 to 1, areal distortion at high latitudes increases dramatically while the distribution of parallels closer to the equator remains constant and mid-latitude land masses, up to latitude 55° N and S, maintain a familiar appearance (Figure 2). Latitudinal compression becomes stronger with aspect ratios less than 0.5. The

lowest aspect ratio (0.3) produces a map that is nearly equal-area with highly flattened polar land masses.

Figures 6–8 show the weighted mean error in areal distortion, the mean angular deformation and the weighted mean error in the overall scale distortion for aspect ratios between 0.3 and 1 for four transformable families of cylindrical projections – equal-area, stereographic, equirectangular, Canters’ transformed Lambert and Miller’s transformed Mercator (described in Section 1.2, with $m = n$) – as well as for selected cylindrical projections with fixed aspect ratios.

Figure 6 shows the weighted mean error in areal distortion. For the aspect-adaptive projection, the weighted mean error in areal distortion increases almost linearly with the aspect ratio. It is interesting to note that almost all compromise cylindrical projections

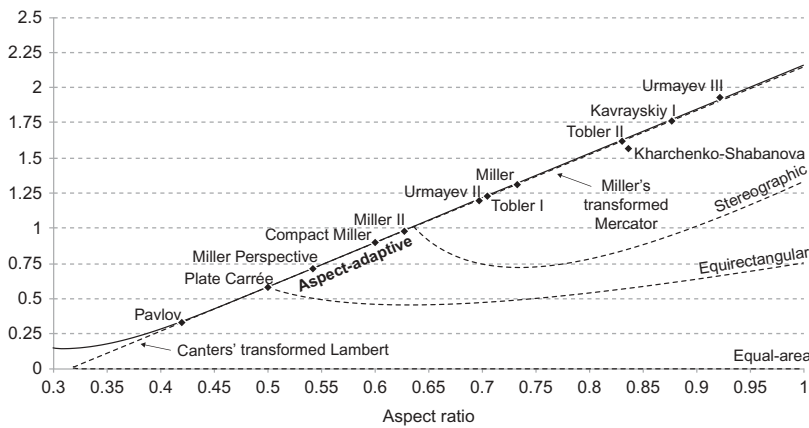


Figure 6. Weighted mean error in areal distortion D_{ar} for the aspect-adaptive cylindrical map projection (solid line) and other selected cylindrical map projections.

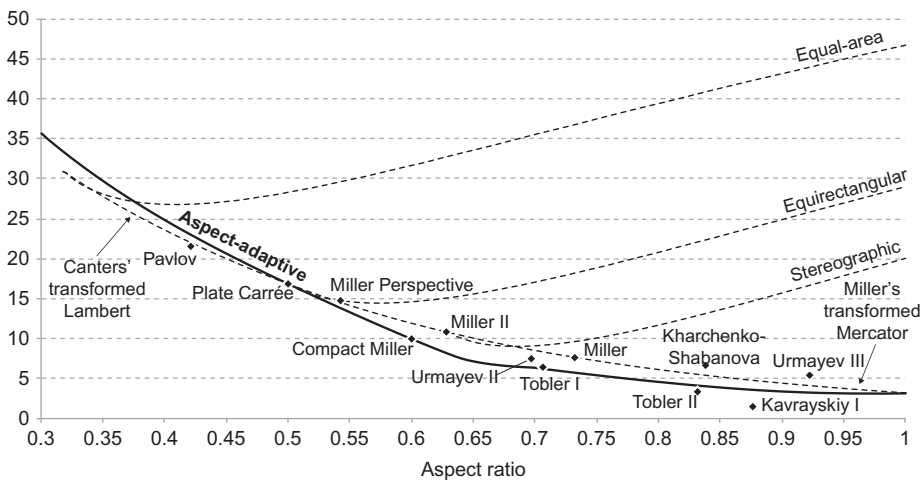


Figure 7. Mean angular deformation D_{an} for the aspect-adaptive cylindrical map projection and other selected cylindrical map projections.

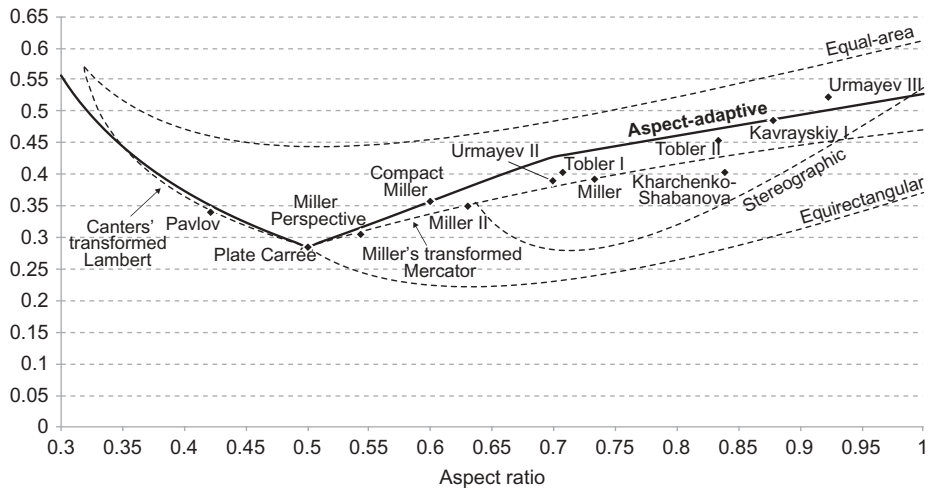


Figure 8. Weighted mean error in the overall scale distortion D_{ab} for the aspect-adaptive cylindrical map projection and other selected cylindrical map projections.

align in Figure 6. The areal distortion of the aspect-adaptive projection for aspect ratios above 0.65 is high when compared to the equirectangular and the stereographic. This is expected because the aspect-adaptive projection increases the spacing of parallels from the equator towards the poles, which is a general characteristic of conformal projections and projections with small angular distortion. The mean angular deformation of the aspect-adaptive projection is low in comparison with other projections. Figure 7 illustrates that the aspect-adaptive projection has less angular distortion than most of the other projections for aspect ratios greater than 0.5. For aspect ratios less than or equal to 0.5, angular distortion is similar to that of other cylindrical projections.

Figure 8 shows that scale distortion of the aspect-adaptive cylindrical projection is similar to all projections with a fixed aspect ratio. The stereographic and equirectangular projection families have lower scale distortion values.

4.2. Evaluation of the Compact Miller projection

When designing the Compact Miller with Flex Projector, the aspect ratio was not initially predefined. Through a process of continuous graphical improvements aiming to optimize the balance between aspect ratio and distribution of parallels, the 0.6 aspect ratio resulted 'naturally'. Larger aspect ratios either disproportionally stretched higher latitudes and wasted canvas space or stretched mid-latitudes, creating an elongated appearance of equatorial land masses, such as Africa, that would appear 'unnatural' to most map users. The 0.6 aspect ratio is close to the golden ratio (0.618) frequently found in classical architecture and art. However, research suggests that world maps with a golden aspect ratio are not innately preferred by map users (Gilmartin 1983).

Figure 5 shows distortion characteristics of the Compact Miller projection with areal and maximum angular distortion isolines and Tissot indicatrices. The equator is the standard parallel without areal or angular distortion. Weighted mean error in areal

distortion, mean angular deformation and weighted mean error in the overall scale distortion indices of the Compact Miller are marked on [Figures 6, 7 and 8](#).

To evaluate the opinions of the general map users and map-makers regarding the Compact Miller projection, we asked 355 map users and 93 map projection experts, cartographers and experienced GIS users to do a pairwise comparison of the Plate Carrée, Braun Stereographic, Mercator, Miller and Compact Miller projections. This online survey was part of the larger user study about user preferences for world map projections (Šavrič *et al.* [in press](#)). Participants were recruited through Amazon's Mechanical Turk, online forums and social networks. Participants were shown all possible 10 pairs created from the set of 5 projections and asked to select the projection they preferred in each pair. Details about the user study survey process, recruiting and statistical analysis can be found in Šavrič *et al.* ([in press](#)).

[Table 4](#) shows the results for general map users. Of the 355 participants, 52% preferred the Plate Carrée to the Compact Miller. The Compact Miller was preferred to the Braun Stereographic by 59% of the participants; 70% preferred the Compact Miller to the Miller and 86% preferred the Compact Miller to the Mercator. [Table 5](#) shows the results for the map projection experts, cartographers and experienced GIS users. Of the 93 participants, 58% preferred the Compact Miller to the Plate Carrée, 70% preferred it to the Braun Stereographic, 83% preferred it to the Miller and 94% preferred it to the Mercator. The overall test of equality (David 1988) was used to determine whether any of the map projections had a significantly different preference compared to

Table 4. Pairwise preference of five cylindrical compromise projections by general map users. The names of the projections are arranged in both rows and columns according to the total scores. Each row shows the percentage of participants that have a preference for the projection in the row to other projections listed in the column. Compact Miller projection is marked in bold.

	1	2	3	4	5
1-Plate Carrée		52%	57%	71%	85%
2-Compact Miller	48%		59%	70%	86%
3-Braun Stereographic	43%	41%		75%	87%
4-Miller Cylindrical	29%	30%	25%		87%
5-Mercator	15%	14%	13%	13%	

Table 5. Pairwise preference of five cylindrical compromise projections by projection experts, cartographers and GIS users. The table has the same ordering and units of measure as [Table 4](#). The Compact Miller projection is marked in bold.

	1	2	3	4	5
1-Compact Miller		70%	58%	83%	94%
2-Braun Stereographic	30%		49%	77%	92%
3-Plate Carrée	42%	51%		62%	88%
4-Miller Cylindrical	17%	23%	38%		94%
5-Mercator	6%	8%	12%	6%	

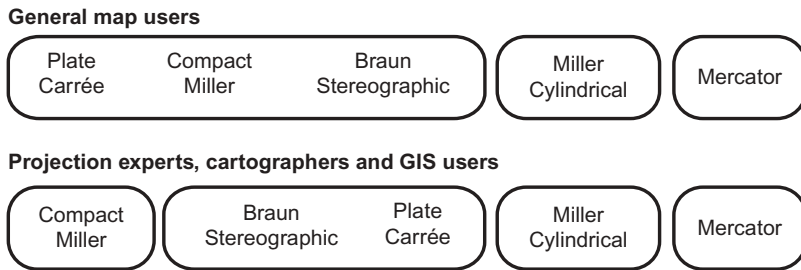


Figure 9. Significant differences in the preferences between the projections in paired comparison test for each participants group. Projections are arranged with the most preferred on the left. Projections circled individually and not grouped with other projections were significantly different in preferences.

all other projections. The test showed that differences in preference existed for both study participants groups: $\chi^2_{4,0.01} = 13.28$ and $D_n = 928.86$ for general map users and $\chi^2_{4,0.01} = 13.28$ and $D_n = 328.52$ for map projection experts, cartographers and experienced GIS users. To determine which graticules were significantly different in preferences, a post hoc analysis was performed with the multiple comparison range test (David 1988). The results, displayed in Figure 9, are arranged with the most preferred projections on the left. Projections circled individually and not grouped with other projections were significantly different in preferences.

General map users preferred the Plate Carrée, Compact Miller and Braun Stereographic projections to the Miller and Mercator projections. There was an almost even split in preference for the Plate Carrée (52%) and Compact Miller (48%) projections. Because the total scores of the Plate Carrée and Compact Miller projections were very close for general map users, we cannot assume that either one of these two projections was preferred more to the other. Experts most frequently preferred the Compact Miller projection to the other four projections. Based on the post hoc analysis with a multiple comparison range test, this preference for the Compact Miller projection is significant (Figure 9).

5. Conclusions

Our advocacy of cylindrical projections with moderate aspect ratios and less exaggerated polar areas parallels the famous Peters controversy. In 1973, Peters reintroduced the Gall projection, an equal-area cylindrical projection with an aspect ratio of 0.636 but flawed by grossly stretched land masses, as a reactionary response to the perceived ‘Eurocentric’ Mercator projection (Sriskandarajah 2003, Vujakovic 2003). The Gall-Peters projection was lauded by popular media and found advocates among international organizations despite objections from respected professional cartographers (e.g. Robinson 1985, 1990). Some cartographic associations, in an attempt to thwart the popularity of the Gall-Peters, took the extreme position of denouncing all rectangular (i.e. cylindrical) world maps (American Cartographic Association *et al.* 1989). Neither position won the argument: the Gall-Peters projection has largely become a historical curiosity, and cylindrical projections dominate today’s online mapping services. The Mercator projection is ubiquitous once again. Our call for using cylindrical projections with moderate aspect ratios, less extreme polar area distortion and recognizable continental shapes is a new attempt at finding

acceptable alternatives to the Mercator projection for world maps. Sociopolitical concerns about the exaggerated size of industrialized countries in the ‘north’ compared to developing tropical countries are a valid argument against using cylindrical world maps with high aspect ratios. Because many other projections show the entire world with considerably less distortion, cylindrical projections should only be used with compelling reasoning. If using compromise cylindrical projections, we recommend avoiding those with aspect ratios less than 0.55 or more than 0.7. If possible, we recommend using the Compact Miller projection with an aspect ratio of 0.6 or one of its close neighbours of the aspect-adaptive cylindrical family with a similar aspect ratio. We believe these projections achieve an acceptable compromise between distortion properties and visual appearance, and the Compact Miller was well received by both general map users and experts in map projections, cartography and GIS.

The aspect-adaptive cylindrical projection family extends the adaptive composite map projection framework by Jenny (2012), which automatically selects projections based on the geographic extent of a map. Adjusting projections to the aspect ratio of maps could be extended in future research to include aspect-adaptive pseudocylindrical projections or world map projections with curved parallels. We hope to see aspect-adaptive projections of various types being used on websites with responsive design and within GIS software.

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